25PY101: Engineering Physics

Module 1 – Unit 2

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Assignment 1: Experiments of Quantum physics

Blackbody radiation

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1 Birth of quantum physics

References:
Section 24.3 Blackbody radiation [Avadhanulu].
Chapter 1 Thermal radiation and Planck's postulate [Eisberg and Resnick]

A body that absorbs light/radiation of any frequency completely is called a **black body**. Since radiation carries energy, the absorbed energy must rise the temperature of the body. Therefore, if the body is under thermal equilibrium, to maintain the temperature, it must emit the radiation back.

1.1 Data extraction

The emission of thermal radiation is given in Figure. 1. In the figure, the variation of energy density $\rho(\lambda, T)$ with wavelength λ is plotted as open circles. The solid line is fit of quantum theory of black body radiation given by Equation. 6.

Problem: Extract the data from the figure for the variation of $\rho(\lambda, T)$ vs λ. [$\star$]
[Hint: Use a scale to draw horizontals and verticals and roughly extract the data corresponding to open circles.]

S.No.	Energy density $\rho(T)$	Wavelength λ

Table 1: Experimental data of $\rho(\lambda, T)$ vs λ at $T = 1595\,\mathrm{K}$.

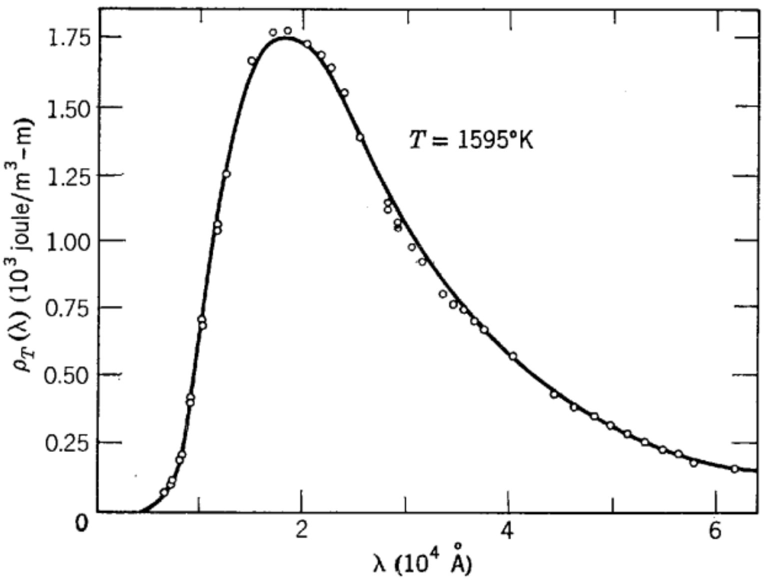

Figure 1-11 Planck's energy density prediction (solid line) compared to the experimental results (circles) for the energy density of a blackbody. The data were reported by Coblentz in 1916 and apply to a temperature of 1595°K. The author remarked in his paper that after drawing the spectral energy curves resulting from his measurements, "owing to eye fatigue it was impossible for months thereafter to give attention to the reduction of the data." The data, when finally reduced, led to a value for Planck's constant of 6.57×10^{-34} joule-sec.

Figure 1: Experimental confirmation of Planck's radiation law. Figure taken from Chapter 1 of Resnick and Eisberg "Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles.

1.2 Ultra-violet catastrophe

The **classical theory** of electromagnetism and thermodynamics predicted the energy density $\rho(\nu, T)$ as

$$\rho(\nu, T) = \frac{8\pi\nu^2 k_B T}{c^3}, \tag{1}$$

where the emission is at temperature T and frequency ν, c is the velocity of light, k_B is the Boltzmann constant. The above expression was derived by Rayleigh and Jeans in 1900. This equation is the basis of **classical theory of black body radiation**.
As a pre-emptive measure for what follows, let us define the characteristic length scale λ_c as

$$\lambda_c = \frac{hc}{k_B T} \tag{2}$$

where h is the famed Planck's constant [1].

1.2.1 Asymptotic analysis

Long wavelength limit: When the ratio of wavelength of radiation to the characteristic length is far greater than unity, it is called the **long wavelength limit**. Therefore, in the long wavelength limit

$$\frac{\lambda}{\lambda_c} \gg 1. \tag{3}$$

Short wavelength limit: When the ratio of wavelength of radiation to the characteristic length is far lesser than unity, it is called the **long wavelength limit**. Therefore, in the short wavelength limit

$$\frac{\lambda}{\lambda_c} \ll 1. \tag{4}$$

The study of behaviour of a function in the limiting cases is called **asymptotic analysis**.
Problem:

(a) Show that the classical expression agrees with experimental data in the long wavelength limit. [⋆⋆]

[Hint: Take the $\ln \rho$ vs λ of Equation. 1 and data.]

(b) In the short wavelength limit, show that the data cannot be fit. This is called **ultra-violet catastrophe**. [⋆]

(c) Discuss the physical consequences of the catastrophe. [⋆]

1.3 Planck's conjecture

Planck conjectured that radiation cannot be absorbed or emitted as any value; instead radiation can only be absorbed or emitted in **quantum/packet** with its energy E and frequency ν related by

$$E = h\nu. \tag{5}$$

[1] Max Planck used the symbol h as a proportionality constant between energy and frequency of photon as a **helper** constant. Hence the symbol h.

The energy density derived by Planck is given by

$$\rho(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}. \tag{6}$$

This equation is the basis of **quantum theory of black body radiation**.
Problem:

(a) Perform the **asymptotic analysis** of Equation. 6 in the long and short wavelength limit. Prove that the quantum theory agrees with the experimental data in both the asymptotic limits. [$\star\star\star$]

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Hint: For long wavelength limit, use the **L'Hopital's rule** which states that if you have $\frac{0}{0}$ form of limit then,

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{f'(x)}{g'(x)},$$

where $f'(x), g'(x)$ are the first derivatives of $f(x), g(x)$ respectively.

Hint: For short wavelength limit,

$$\lim_{x\to\infty} \exp(x) = \infty, \qquad \lim_{x\to\infty} x^n \exp(-x) = 0, \ \forall n > 0$$

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(b) Show that the expression for the wavelength with the **maximum energy density** $\lambda_{\max}$ at a given temperature T is given by [$\star\star$]

$$\lambda_{\max} T = \text{const} \tag{7}$$

This is called the **Wien's displacement law** and the constant is called Wien's constant. The experimental value of Wien's constant is $2.898 \times 10^{-3}\,\mathrm{m\,K}$.

[Hint: To find the peak in a function, you must equate its dervative to zero.]

(c) Find the expression for Wien's constant from Equation. 7. [$\star$]

(d) As the temperature of blackbody increases, does $\lambda_{\max}$ red-shift or blue-shift [2]? [$\star$]

(e) If we assume that stellar surfaces behave like blackbodies we can get a good estimate of their temperature by measuring $\lambda_{\max}$. For the sun, $\lambda_{\max} = 5100\,\text{Å}$ (green light) i.e. the greatest part of its radiation lies within the visible region of the spectrum. This suggests over the ages of human evolution, our eyes have adapted to the sun to become most sensitive to those wavelengths which it radiates most intensely. Find the surface temperature of sun. [$\star$]

[2] When the wavelength increases, we use the expression "wavelength has red-shifted"; when it decreases, we use the expression "wavelength has blue-shifted". The reason being red wavelength is bigger than that of blue.

(f) Show that the specific power [3] $\mathcal{P}$ is given by integrating the energy density over the entire spectrum of wavelengths

$$\mathcal{P} = \int_0^\infty \rho(\nu)\,\mathrm{d}\nu \propto T^4, \qquad \Rightarrow \qquad \mathcal{P} = \sigma T^4 \tag{8}$$

This is called the **Stefan-Boltzmann law** and the constant of proportionality σ is called the Stefan's constant. The experimental value of Stefan's constant is $5.67 \times 10^{-8}\,\mathrm{W/m^2K^4}$. [$\star\star\star\star$]

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Hint: Use the dimensionless variable $x = \frac{h\nu}{k_B T}$ for integration, so that

$$\mathrm{d}\nu = \frac{k_B T}{h}\mathrm{d}x$$

Hint:

$$\int_0^\infty \frac{x^3}{e^x - 1}\,\mathrm{d}x = \Gamma(4)\zeta(4),$$

where $\Gamma(x)$ is called **Gamma function** and $\zeta(x)$ is called **Riemann zeta function**. These are frequently occuring functions in physics.

Gamma function is the generalization of factorials and is given by

$$\Gamma(x) = \int_0^\infty x^{n-1} e^{-x}\,\mathrm{d}x.$$

For positive integers,

$$\Gamma(n) = (n-1)!$$

Riemann zeta function is given by

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1}\,\mathrm{d}x.$$

In this derivation, we are interested in

$$\int_0^\infty \frac{x^3}{e^x - 1}\,\mathrm{d}x. = \Gamma(4)\zeta(4).$$

The values of $\Gamma(4)$ and $\zeta(4)$ can be shown to be

$$\Gamma(4) = 3! = 6, \qquad \zeta(4) = \frac{\pi^4}{90}.$$

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(g) Find the expression for Stefan's constant from Equation. 8. [$\star$]

(h) As the temperature of blackbody increases, does the energy density $\rho(\lambda)$ at $\lambda_{\max}$ increase or decrease? [$\star$]

[3] Specific power is defined as the total energy per unit area per unit time.

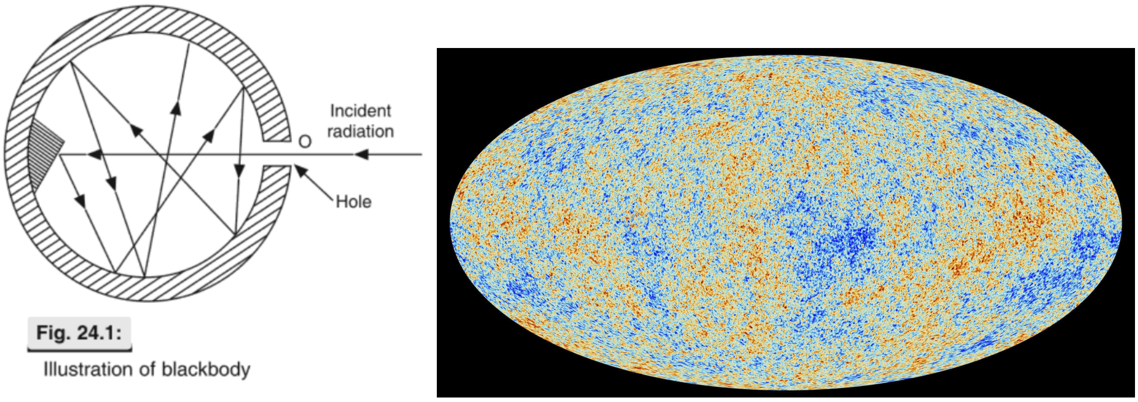

Figure 2: (a) A table top black body. Taken from Avadhanulu.(b) Universe as a **perfect** black body as inferred from the temperature map of the cosmic microwave background measured by Planck spacecraft. Taken from Wikipedia.

(i) Using Stefan's law and Wien's law, plot the energy spectrum of black body radiation at different temperatures (say 4000 K, 5000 K, 6000 K, and 7000 K) qualitatively? [$\star\star$]

(j) From the experimentally observed values of Wien's constant and Stefan's constant, find the values of the Planck's constant and Boltzmann's constant. Assume you know the value of speed of light. This was done by Planck on December 14, 1900 in his seminal paper, "On the Theory of the Energy Distribution Law of the Normal Spectrum". This date is considered as the **birthday of quantum physics**. [$\star\star$]

1.4 Cosmic microwave background

A perfect black body is an idealization. The closest that we can come to a perfect black body in a laboratory is a body with a pinhole, so that radiation can be incident as shown in Figure. 2(a). However, the universe can also be considered as a black body that is emitting and absorbing photons between its components. The only difference between the laboratory version and the universe is that in the former, the observer is outside the body whereas in the case of universe, the observer is inside the black body.

In 1965, radio astronomers Allan Penzias and Robert Wilson accidentally detected microwave radiation that appears to come from every direction of the sky. The data they obtained was fit to the Planck's black body law and fit was uncharacteristically perfect. To this date, this is the most accurate confirmation of Planck's law and the universe can be considered as a nearly perfect black body.

Problem: The temperature of the universe is experimentally measured to be

$$T_{\text{universe}} = 2.725\,48\,\text{K} \pm 0.000\,57\,\text{K}.$$

The temperature variation is plotted in Figure. 2(b) as a colormap with red indicating the maximum and blue indicating the minimum. Calculate the wavelength corresponding to maximum intensity. [$\star$]

End of Assignment