1. Solar cell **[10 M]**

a) Write the equations for $I - V$ characteristics and $P - V$ characteristics of pn junction solar cell. [BT 1][2 M]

b) Derive the equation to determine the operating voltage for maximum power generation V_m in terms of the photocurrent I_L and reverse saturation current I_s. [BT 3][4 M]

c) The reverse saturation current density for a solar cell pn junction is $3.6 \cdot 10^{-11} A/cm^2$. What is the photocurrent density required to generate open circuit voltage of $0.60\ V$. [BT 4][4 M]

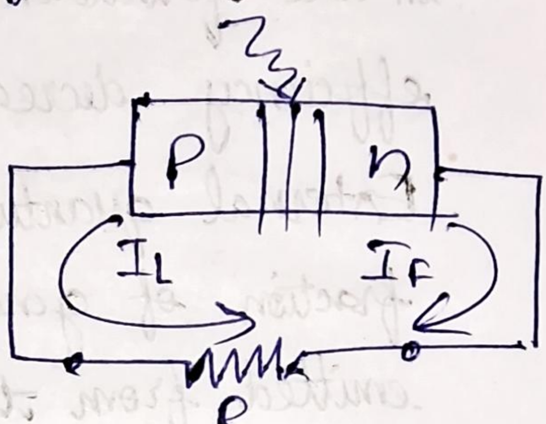
PART - B

① Solar cell

a) Write the equations for I-V & P-V characteristics of pn junction solar cell.

Solution:

$$I = I_L - I_F$$



$$I = I_L - I_S\left[\exp\left(\frac{eV}{K_BT}\right) - 1\right]$$

and $P = VI$ $\qquad V = IR$

$$P = V\left[I_L - I_s\left\{\exp\left(\frac{eV}{K_BT}\right) - 1\right\}\right]$$

So we have P(V). and we have conditions for optimization.

b) Derive the equation for determining the operating voltage for maximum power generation V_m $\bar{c}$ I_L and I_s.

$$\frac{dP}{dV} = 0 \Leftrightarrow I_L - I_s\left\{\exp\left(\frac{eV}{K_BT}\right) - 1\right\}$$

$$+ V\left[-I_s\frac{e}{K_BT}\exp\left(\frac{eV}{K_BT}\right)\right] = 0$$

$$\Leftrightarrow I_L - I_s\left\{\exp\left(\frac{V}{V_t}\right) - 1\right\} - I_s\frac{V}{V_t}\exp\left(\frac{V}{V_t}\right) = 0$$

$$\Leftrightarrow \exp\left(\frac{V}{V_t}\right)\left\{I_s + I_s\frac{V}{V_t}\right\} = I_L + I_s$$

$$\Leftrightarrow \exp\left(\frac{V}{V_t}\right)\left(1 + \frac{V}{V_t}\right) = 1 + \frac{I_L}{I_s}$$

c) The reverse saturation current density for a solar cell pn junction is $3.6 \cdot 10^{-11}\ A/cm^2$. What is the photocurrent density required to generate open circuit voltage of 0.60 V?

Solution:

At open circuit conditions

$$I = 0 \Rightarrow I_L = I_s\left[\exp\left(\frac{V_{oc}}{V_t}\right) - 1\right]$$

$$\Rightarrow \exp\left(\frac{V_{oc}}{V_t}\right) = 1 + \frac{I_L}{I_s}$$

$$\Rightarrow \frac{V_{oc}}{V_t} = \ln\left(1 + \frac{I_L}{I_s}\right)$$

$$\Rightarrow V_{oc} = V_t \ln\left(1 + \frac{I_L}{I_s}\right)$$

$$0.60 = 0.0259 \ln\left(1 + \frac{I_L}{I_s}\right)$$

$$\Rightarrow \ln\left(1 + \frac{I_L}{I_s}\right) = \frac{0.60}{0.0259}$$

$$\ln\left(1 + \frac{I_L}{I_s}\right) = 23.16$$

$$\Rightarrow 1 + \frac{I_L}{I_s} = \exp(23.16)$$

$$1 + \frac{I_L}{I_s} = 1.14 \cdot 10^{10}$$

$$I_L \simeq I_s \cdot 1.14 \cdot 10^{10}$$

$$= 3.6 \cdot 10^{-11} \cdot 1.14 \cdot 10^{10}$$

$$= 4.1 \cdot 10^{-1}\ A/cm^2$$

$$= 0.41\ A/cm^2$$

2. n_0p_0 product in extrinsic semiconductor **[10 M]**

a) Design a new semiconductor material. It has to be p type and doped with $N_a = 5 \cdot 10^{15} cm^{-3}$ acceptor atoms. Assume complete ionization and assume $N_d = 0$. The effective density of states functions are $N_c = 1.2 \cdot 10^{19} cm^{-3}$ and $N_v = 1.8 \cdot 10^{19} cm^{-3}$ at $T = 300K$. The requirement is that the hole concentration must not exceed $5.08 \cdot 10^{15} cm^{-3}$ at $T = 350K$. What is the minimum bandgap energy required in this material? You can assume the n_0p_0 product rule is valid for extrinsic semiconductor also. [Hint: Apply the n_0p_0 product rule on the charge neutrality condition $n_0 + N_a^- = p_0 + N_d^+$.] [BT 6][10 M]

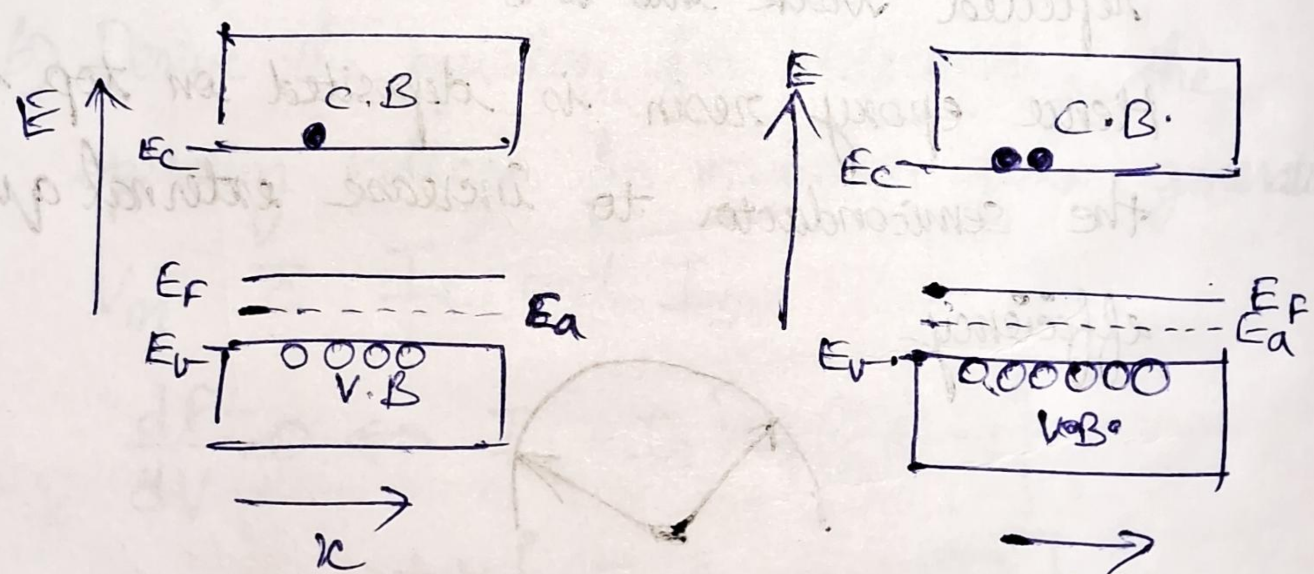
② $n_o p_o$ product

a) Design a new semiconductor. It has to be p-type and doped with $N_a = 5\cdot10^{15}\ cm^{-3}$ acceptor atoms. Assume complete ionization and assume $N_d = 0$. The effective density of states functions are $N_c = 1.2\cdot10^{19}\ cm^{-3}$ and $N_v = 1.8\cdot10^{+19}\ cm^{-3}$ at $T = 300K$.

The requirement is that the hole concentration must not exceed $5.08\cdot10^{15}\ cm^{-3}$ at $T = 350K$. What is the minimum bandgap energy required in this material?

Solution:

@ T=300K @ T=350K



As temperature increases, thermally generated electron-hole pairs increase in addition to the majority ~~carriers~~ holes due to acceptors.

~~@ T=300 let~~ ~~n_0~~

@ T=350K,

let n_0 - electron (minority)

& p_0 - hole (majority) be in ptype

Now acceptors contribute holes.

After ionization, they are ionized to N_a^-

In p type,

charge neutrality enforces

total negative charge = total positive charge

$$n_0 + N_a^- = p_0$$

Now $n_0 p_0$ product rule relates n_0 to p_0

$$\Rightarrow \quad n_0 p_0 = n_i^2$$, n_i is the intrinsic conc @ T=350

$$\Rightarrow \quad n_0 = \frac{n_i^2}{p_0}$$

$$\therefore \quad \frac{n_i^2}{p_0} + N_a^- = p_0$$

Due to complete ionization, $N_a^- = N_a$
$= 5\cdot 10^{15}\ cm^{-3}$

so that intrinsic conc is given by

$$\frac{n_i^2}{p_0} = p_0 - N_a$$

Now $p_0 = 5.08 \cdot 10^{15}$

so that $n_i^2 = (5.08-5)\cdot 10^{15} \cdot 5.08 \cdot 10^{15}$

$$\Rightarrow n_i^2 = 0.4064 \cdot 10^{30}$$

Intrinsic concentration varies with temperature as

$$n_i^2 = N_C N_V \exp\left(-\frac{E_g}{K_B T}\right)$$

$\because$ N_C & N_V varys as $T^{3/2}$ invidually

$$\therefore \; n_i^2 \propto T^3 \exp\left(-\frac{E_g}{K_B T}\right)$$

$$\therefore \; \frac{n_i^2(@350K)}{T_{350}^3 \exp\left(\frac{-E_g}{K_B T_{350}}\right)} = \frac{n_i^2(@300K)}{T_{300}^3 \exp\left(-\frac{E_g}{K_B T_{300}}\right)}$$

$$= \frac{N_C N_V \cancel{\exp\left(\frac{-E_g}{K_B T_{300}}\right)}}{T_{300}^3 \cancel{\exp\left(-\frac{E_g}{K_B T_{300}}\right)}}$$

$$\Rightarrow \exp\left(\frac{-E_g}{K_B T_{350}}\right) = \left(\frac{T_{300}}{T_{350}}\right)^3 \frac{n_i^2(@350K)}{N_C N_V}$$

$$\Rightarrow \exp\left(\frac{-E_g}{K_B T_{350}}\right) = \left(\frac{300}{350}\right)^3 \frac{0.4064 \cdot 10^{30}}{1.2 \cdot 10^{19} \cdot 1.8 \cdot 10^{19}}$$

$$\Rightarrow \exp\left(\frac{-E_g}{K_B T_{350}}\right) = 0.118 \cdot 10^{-8}$$

$$\Rightarrow \cancel{-}\frac{E_g}{K_B T_{350}} = \cancel{-}\,20.55$$

$\therefore E_g = 0.62\,eV$ $\quad 30$ $\quad -38$

$$\Rightarrow E_g = K_B T_{350} \cdot 20.55 = 0.0259 \left(\frac{350}{300}\right) \cdot 20.55$$

3. Blackbody radiation and optical absorption **[10 M]**

a) The Planck's blackbody radiation law is given by

$$I(\nu) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

where $I(\nu)$ is the irradiance at a given frequency ν and temperature T. Find the expression for wavelength λ_m corresponding to maximum irradiance in terms of temperature T. [BT 3][4 M]

b) An alien ship crash landed onto Earth's surface. From the debris, the solar panel used for power generation is found to have a band gap of 5 eV. Estimate the temperature of the alien Sun. [BT 5][4 M]

c) Is the solar panel useful for applications on Earth? Why or why not? [BT 2][2 M]

3 Black body radiation and optical absorption

a) Planck's blackbody radiation law is

$$I(\nu) = \frac{2h\nu^{3}}{c^{2}} \cdot \frac{1}{\exp\left(\frac{h\nu}{K_B T}\right) - 1}$$

where I is the irradiance at a given frequency ν and temperature T.

Find the expression for wavelength λ_m corresponding to maximum irradiance $\bar{c}$ T.

Solution

For Wien's displacement law derivation

it is useful to work with irradiance as a function of wavelength

$$\boxed{I(\nu)\,d\nu = -I(\lambda)\,d\lambda}$$

~~Now~~ $\because$ sign indicates changes in ν and λ are opposite to each other

$$I(\lambda) = -I(\nu)\frac{d\nu}{d\lambda}$$

But $\nu = \frac{c}{\lambda}$

$$\Rightarrow \frac{d\nu}{d\lambda} = c\frac{d}{d\lambda}\left(\frac{1}{\lambda}\right) = \frac{-c}{\lambda^2}$$

$$\therefore I(\lambda) = \frac{c}{\lambda^2} I(\nu)$$

$$= \frac{2hc}{c^2}\frac{c^3}{\lambda^3\lambda^2}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1}$$

$$\boxed{I(\lambda) = \frac{2hc^2}{\lambda^5}\cdot\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1}}$$

Now λ_{max} is given by

$$\left.\frac{dI}{d\lambda}\right|_{\lambda_{max}} = 0 \Leftrightarrow \frac{-5}{\lambda^6}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1} + \frac{1}{\lambda^5}\cdot\frac{-1}{\left[\exp\left(\frac{hc}{\lambda k_B T}\right)-1\right]^2}\frac{hc}{k_B T}\exp\left(\frac{hc}{\lambda k_B T}\right)\frac{-1}{\lambda^2} = 0$$

$$\Leftrightarrow -5\left[\exp\left(\frac{hc}{\lambda k_B T}\right)-1\right] + \frac{hc}{\lambda k_B T}\exp\left(\frac{hc}{\lambda k_B T}\right) = 0$$

If $x = \frac{hc}{\lambda_{max} K_B T}$

then

$$-5(e^x - 1) + x e^x = 0$$

$$5(1-e^x) + x e^x = 0$$

$$5(e^{-x} - 1) + x = 0$$

$$x = 5(1 - e^{-x})$$

In the short wavelength limit

$$x \gg e^{-x}$$

$$\Rightarrow \quad x \simeq 5$$

$$\Rightarrow \quad \frac{hc}{\lambda_{max} K_B T} = 5$$

$$\Rightarrow \quad \lambda_{max} T = \frac{hc}{5 K_B}$$

$$= \frac{6.625 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{5 \cdot 1.38 \cdot 10^{-23}}$$

$$\boxed{\lambda_{max} T = 2.88 \cdot 10^{-3}\, mK}$$

(b) Alien ship has crash landed onto Earth's surface. From the debris, solar panel used for power generation is found to have a band gap of 5 eV. Estimate the temperature of alien Sun.

Solution :

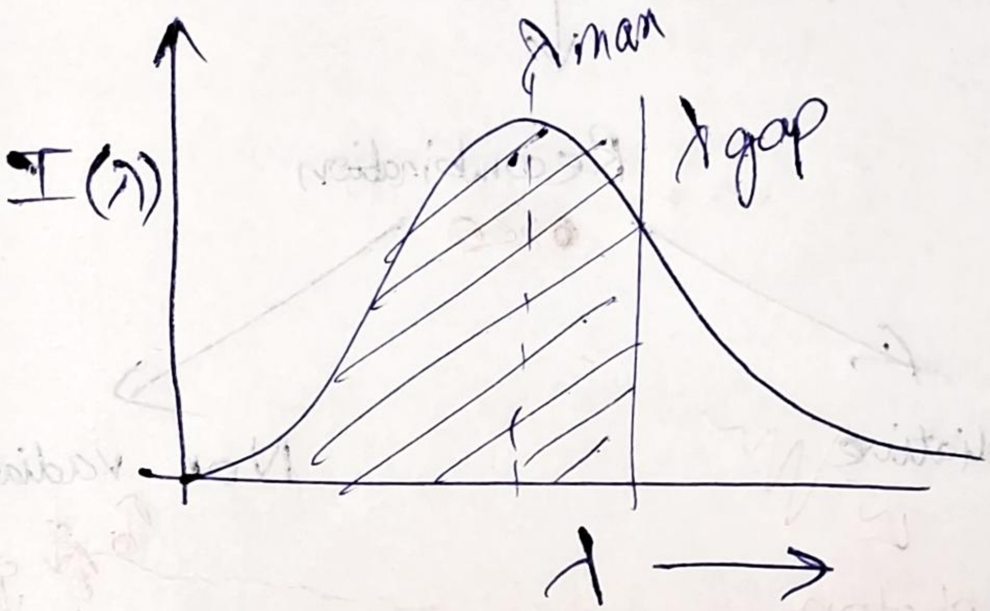
λ_{gap} of solar panel material is

$$(\mu m)\ \lambda_{gap} = \frac{1.24}{E_{gap}\,(eV)}$$

$$= \frac{1.24}{5}$$

$$\lambda_{gap} = 0.248\ \mu m$$

Now if the alien Sun can be assumed to be a black body, its radiation spectrum should look like .



Now if the aliens are logical beings they should be using solar panel whose λ_{gap} covers most of the spectrum radiation

~~So~~ ~~λ_{gap}~~ If λ_{max}^{alien} is the maximum irradiance wavelength, then

$$\lambda_{max}^{alien} \leq \lambda_{gap}$$

$$\Rightarrow \lambda_{max}^{alien} \le 0.248\,\mu m$$

Now the λ_{max}^{alien} is related to Wiens displacement law by:

$$\lambda_{max}^{alien}\, T_{alien} = 2.88\cdot 10^{-3}\,mK$$

$$\Rightarrow T_{alien} = \frac{2.88\cdot 10^{-3}\,mK}{\lambda_{max}^{alien}}$$

$$\Rightarrow T_{alien} \ge \frac{2.88\cdot 10^{-3}}{0.248\cdot 10^{-6}}$$

$$= 11.6\cdot 10^{3}\,K$$

$$\boxed{T_{alien} = 11,600\ K}$$

(c) Is the solar panel useful for applications on Earth? Why or why not?

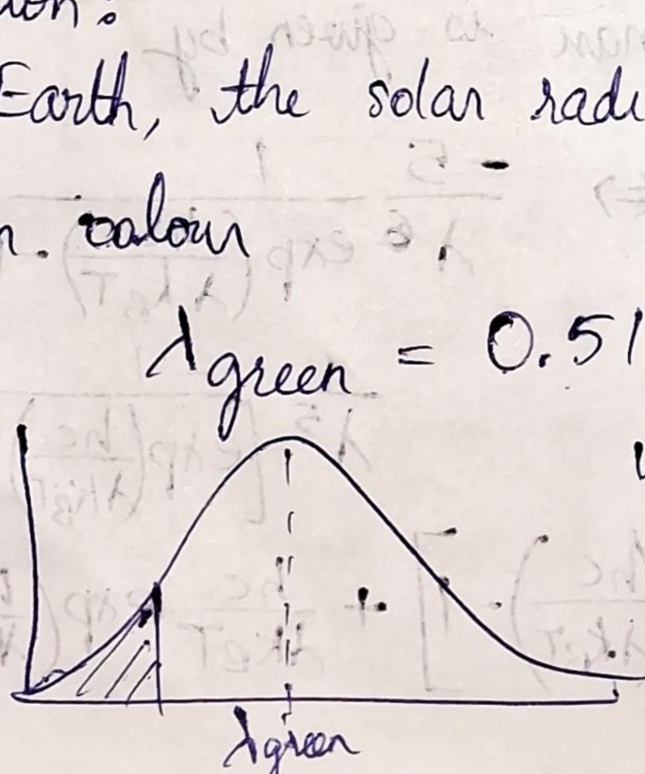
Solution:

At Earth, the solar radiation peaks at green colour

$$\lambda_{green} = 0.51\,\mu m$$

whereas $\cancel{\lambda_{max}^{alien}}$

$$\lambda_{gap} = 0.248\,\mu m$$



So very small portion of spectrum can be absorbed by the solar panel. Thus, it would not be a very efficient solar cell.

END OF PART-B