Engineering Physics (2025)
Course code 25PY101
Unit 2: Quantum mechanics

Course Instructor:
Dr. Sreekar Guddeti
Assistant Professor in Physics
Department of Science and Humanities
Vignan's Foundation for Science, Technology and Research

October 25, 2025

Unit 2 Plan

1. Introduction to QM
2. Dual nature of radiation
3. de Broglie's concept of matter waves
4. Heisenberg's uncertainty principle
5. Schrödinger's time dependent wave equation
6. Particle in a 1D box

Unit 2 Plan

$$\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} + \frac{2m}{\hbar^2}\big(E - V(x)\big)\psi(x) = 0 \qquad \Psi(x,t) = \psi(x)e^{-j\omega t}$$

- The aim is to solve for the time independent wave function $\psi(x)$ for a given potential $V(x)$.
- The potential functions that are useful to describe commonly occurring physical systems are

$V(x)$	Name	Physical systems
0	Constant potential	Particle in free space
$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{cases}$	Infinite potential well	Ideal atom
$V(x) = \begin{cases} 0 & -\frac{a}{2} < x < \frac{a}{2} \\ V_0 & \text{otherwise} \end{cases}$	Finite potential well	Work function
$V(x) = \begin{cases} V_0 & -\frac{a}{2} < x < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$	Finite barrier	Nuclear fission
		Scanning Tunneling Micros

Particle in free space

$V(x)$: Constant potential

$$\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0 \qquad \Psi(x,t) = \psi(x)e^{-j\omega t}$$

- Let us define the wavevector k as

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

- The solution of differential equation can be written as

$$\psi(x) = Ae^{jkx} + Be^{-jkx}$$

- If we use the Euler's formula

$$e^{jx} = \cos x + j\sin x$$

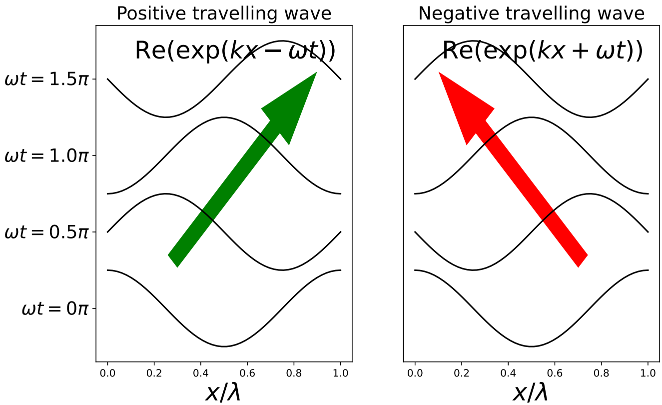
then the solution can be written also as

$$\psi(x) = A'\cos kx + B'\sin kx$$

$V(x)$: Constant potential

- The solutions are **travelling** waves.
- The energy E can be any positive real number and is related to the wavevector k by

$$E = \frac{\hbar^2 k^2}{2m}$$



Constant potential: problems

Problem

An electron in free space is described by a plane wave given by $\Psi(x,t) = A\exp(j(kx - \omega t))$. *If* $k = 8 \times 10^{8}\,\mathrm{m^{-1}}$ *and* $\omega = 8 \times 10^{12}\,\mathrm{rad\,s^{-1}}$, *determine*

1. *phase velocity and wavelength of the plane wave*
2. *momentum and kinetic energy (in* eV*)*

Repeat the steps for $k = -1.5 \times 10^{9}\,\mathrm{m^{-1}}$ *and* $\omega = 1.5 \times 10^{13}\,\mathrm{rad\,s^{-1}}$.

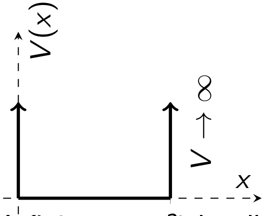
Problem

Determine the wave number, wavelength, angular frequency, and period of wavefunction that describes an electron traveling in free space at a velocity of (a) $5 \times 10^{6}\,\mathrm{cm\,s^{-1}}$.

Particle in a 1D box

$V(x)$: Infinite potential

$$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{cases}$$



Infinite potential well

- Consider a particle in a 1-dimensional infinite potential well.
- The well can be considered as a box.

Outside infinite box

$$\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} + \frac{2m}{\hbar^2}(E - \infty)\psi(x) = 0$$

Inside infinite box

$$\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

$V(x)$: Infinite potential well

Outside infinite box

- There is no wavefunction!

$$\psi(x) = 0$$

- Upon applying boundary conditions

Inside infinite box

- The wavefunction is

$$\psi(x) = A\cos kx + B\sin kx$$

Left B.C.

$$\psi(x=0) = 0$$
$$A\cos kx = 0$$
$$\Rightarrow A = 0$$

Right B.C.

$$\psi(x=a) = 0$$
$$B\sin ka = 0$$
$$\Rightarrow \sin ka = 0$$

$V(x)$: Infinite potential well

- The equation $\sin ka = 0$ is valid if
$$ka = n\pi$$
where n is a positive integer, or $n = 1, 2, 3, \ldots$. The parameter is referred to as quantum number. In terms of the quantum number, the wavevector is given by
$$k = \frac{n\pi}{a}$$
- The prefactor B can be found by applying the **normality condition** of wave function.
$$\int_0^a |B \sin kx|^2 \, \mathrm{d}x = 1, \qquad \Rightarrow \qquad \int_0^a B^2 \sin^2 kx \, \mathrm{d}x = 1$$
$$\therefore B = \sqrt{\frac{2}{a}}$$
- Therefore, the time independent wave function is given by
$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$V(x)$: Infinite potential well

- Recall that the wavevector k is related to the energy E by

$$k = \sqrt{\frac{2mE}{\hbar^2}}, \qquad \Rightarrow \qquad E = \frac{\hbar^2 k^2}{2m}$$

- Therefore, the energy is given by

$$E = E_n = \frac{n^2 \hbar^2 \pi^2}{2ma^2}$$

Problem

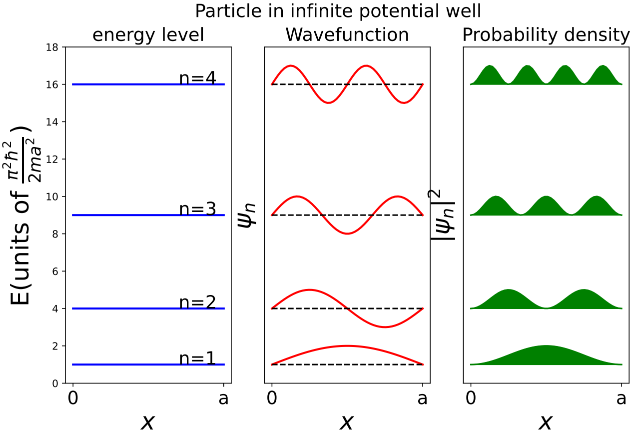
Electron in an infinite potential well of width 5 Å. *Calculate the first three energy levels.*

Key Insight

The energy of particle is **quantized** i.e. it can only have particular **discrete** values.

$V(x)$: Infinite potential well

- The solutions are **standing** waves.



Orthogonality condition

- Wave function associated with an energy level is normalized.
- In addition to normality, wavefunctions $\psi_m(x), \psi_n(x)$ corresponding to any two levels m, n of infinite potential well are orthogonal to each other i.e.

$$\int_0^a \psi_m^*(x)\psi_n(x)\,\mathrm{d}x = 0.$$

  This is called the **orthogonality condition**.

Theorem

Prove the orthogonality condition for solutions of infinite potential well.

1D infinite well: problems

Problem

1. *A particle with mass of* 15 mg *is bound in a 1D infinte potential well that is* 1.2 cm *wide.*
2. *If the energy of the particle is* 15 mJ, *determine the value of n for that state.*
3. *What is the energy of the $(n+1)$ state?*
4. *Would quantum effects be observable for this particle?*