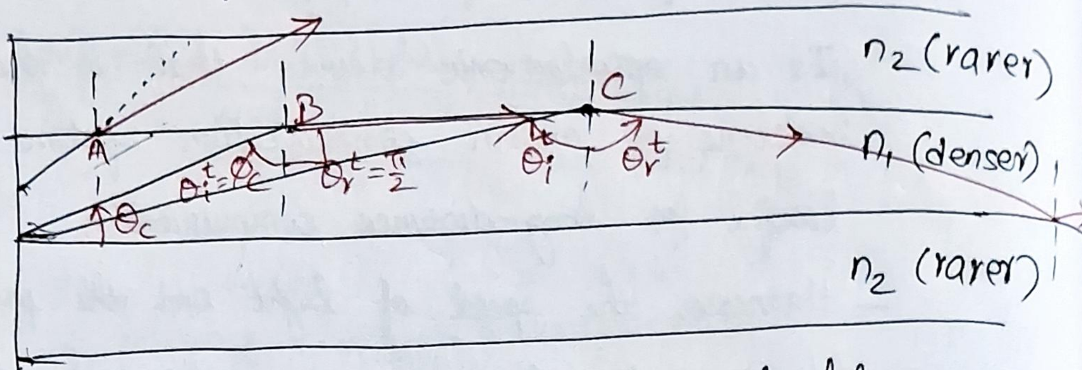
Optical fibre - conditions



- Light is at one end of the optical fibre.
- Not all the light can be contained within the light pipe (optical fibre)

At the "top" interface

- If the angle of incidence is less than critical angle of incidence, the light refracts and escapes the fibre (case A)
- If the angle of incidence is equal to ~~and~~ critical angle ~~of~~ incidence the light grazes the interface and is contained within the core (case B)
- If the angle of incidence is greater than critical angle of incidence, the light reflects back into the core (case C)

At the "bottom" interface

- After first reflection, the light is incident at "bottom" interface. We again encounter the condition of denser to rarer medium
- However, from arguments of plane geometry, it can be shown that

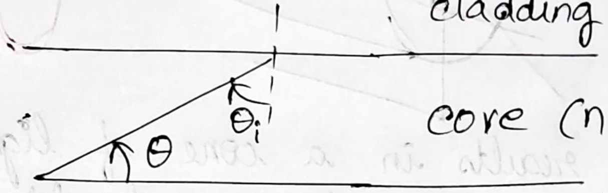
$$\theta_i^b = \theta_r^t > \phi_c$$

So total internal reflection occurs again.

Upon extending this analysis to ad infinitum, it is proved that light is contained within the fibre for all angle of incidences greater than critical angle of incidence.

Critical angle of propagation

With reference to the core-cladding, we define the angle of propagation as the angle between incident ray and core-cladding interface.



$$\theta = \frac{\pi}{2} - \theta_i$$

Since $\theta_i \geq \phi_c$ is contained within the fibre

$\theta = \frac{\pi}{2} - \theta_i \leq \frac{\pi}{2} - \phi_c$ is the conditioned for angle of propagation.

If we define the critical angle of propagation

$$\theta_c = \frac{\pi}{2} - \phi_c$$

then for all $\theta \leq \theta_c$, the light is contained/confined.

Critical angle of incidence is given by Snell's law

If the refractive inden of core is n_1

refractive inden of cladding is n_2

then $n_1 > n_2$

and for case (A)

$$\frac{\sin \theta_i^t}{\sin \theta_r^t} = \frac{n_2}{n_1}$$

for case (B) $\theta_r^t = \frac{\pi}{2}$, $\theta_i^t = \phi_c$

so

$$\frac{\sin \phi_c}{\sin \frac{\pi}{2}} = \frac{n_2}{n_1}$$

$$\Rightarrow \sin \phi_c = \frac{n_2}{n_1}$$

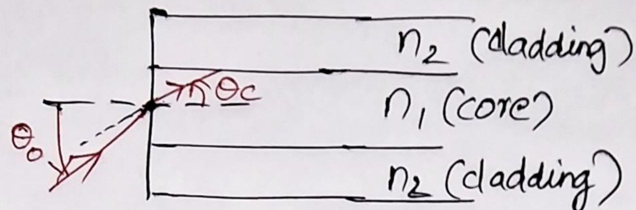
$$\Rightarrow \boxed{\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

Therefore the critical angle of propagation is given by

$$\boxed{\theta_c = \frac{\pi}{2} - \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

Acceptance angle

Before the light enters the core medium, it has encounter the interface ~~or~~ between the core and the ambient medium. If the refractive indexe of ambient medium is n_3, then



for entering the core medium, the refractive index of ambient must be ~~rarer~~ lesser than refractive index of core

For ~~convience~~ convenience, let us consider $n_3 = 1$.

Therefore the angle of incidence at the ambient – core interface ~~is~~ needs to be less than ~~a~~ ~~crit~~ the so called acceptangle angle so that refracted light is within the critical angle of propagation.

At the acceptance angle of incidence θ_0

$$\frac{\sin\theta_0}{\sin\theta_c} = \frac{n_1}{n_3} = n_1$$

$$\Rightarrow \sin\theta_0 = n_1 \sin\theta_c$$

$$= n_1 \sin\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{n_2}{n_1}\right)\right)$$

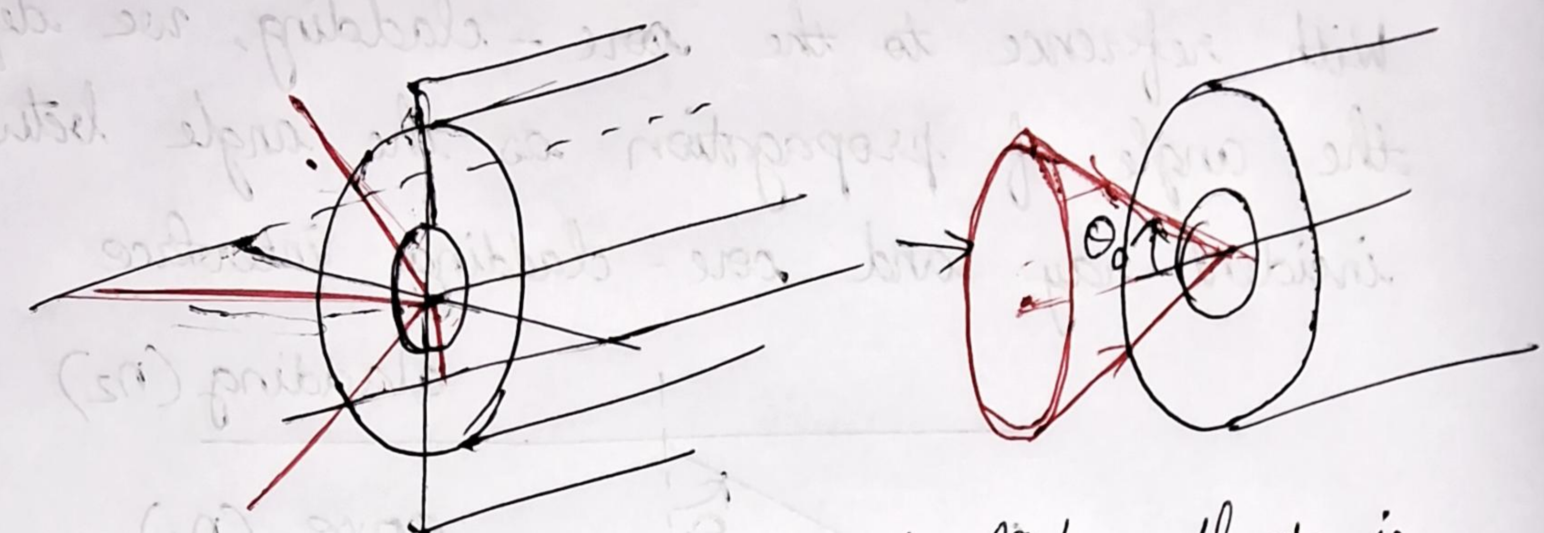
$$= n_1 \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\Rightarrow \boxed{\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2}}$$

Acceptance cone

We have limited our previous analysis to the "top" and "bottom" interface. However due to the cylindrical nature of the optical fibre, the analysis is applicable to all the planes passing through the axis.



This results in a cone of light that is accepted into the optical fibre. This is called acceptance cone and is defined by its acceptance angle